REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
8/12/97

3. REPORT TYPE AND DATES COVERED
7/1/94 - 9/30/97

4. TITLE AND SUBTITLE
Applications of Geometric Invariants to Computer Vision - AASERT Grant

5. FUNDING NUMBERS
F49620-94-1-0280
AFOSR-TR-97
0663

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8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Air Force Office of Scientific Research
Bolling AFB, DC 20332-6448

nm

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES
The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*
This is an AASERT grant that is a supplementary award to the parent AFOSR Grant FR9620-94-1-002. The objective was to train a graduate student who must be a U.S. citizen. We have achieved this objective and the student will be awarded a Ph.D. in Computer Science in August 1997. Also, the student published several papers in refereed journals and gave oral presentations in two international conferences. We developed a method for object recognition by extracting conics from digitized images and computing their invariants. The proposed method was extensively tested under noisy conditions and the results were highly satisfactory. We also investigated the geometrical properties of the Legendre transform that are of interest in image understanding applications. Finally, we developed a matching algorithm for space curves, which reduces the problem to finding the eigenvalues of a 4-dimensional matrix.

14. SUBJECT TERMS

15. NUMBER OF PAGES
6

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTR
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-8
Prescribed by ANSI Std. 239-18
298-102

Final Report
AFOSR Grant F49620-94-1-0280

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August 11, 1997

- Title of project: "Applications of Geometric Invariants to Computer Vision" [1]
- Program manager: **Dr. Jon A. Sjogren**
- Total duration of project: July 1, 1994 — June 30, 1997.

[1] AASERT Grant linked to AFOSR Grant F49620-94-1-0029

1 Status of Effort

This is an AASERT grant that is a supplementary award to the parent AFOSR Grant FR9620-94-1-002. The objective of the AASERT grant was to train a graduate student who must be a US citizen. We were highly successful in our objectives – the graduate student Douglas E. Heisterkamp successfully completed all the requirements for a Ph.D. and also published several journal papers (see the list at the end of the report). Also, he gave oral presentations in two international conferences, one in Minneapolis, and the other one in the Vienna, Austria. The PI made a considerable effort in training the AASERT graduate student in the area of the project.

The accomplishments given below overlap with some of the accomplishments of the parent AFOSR grant of which the present grant was a supplementary.

2 Accomplishments

We have constructed mutual invariants of families of coplanar conics. These invariants are compared with the use of invariants of two conics and a case is presented where the proposed invariants have a greater discriminating power than the previously used invariants. The use of invariants for two conics is extended to any number of coplanar conics. A lambda-matrix is associated with each family of coplanar conics. The use of lambda-matrices is extended from the single variable polynomial to multi-variable polynomials. The Segre characteristic and other invariants of the lambda-matrix are used as invariants of the family of conics.

Conics are widely recognized in the study of machine vision as the most fundamental image features next to lines. Many natural and man-made objects have circular shapes, and in addition, many other curves can be approximated by conics. Arbitrary planar shapes can be represented by a set of coplanar conics. There has been much work done in recognizing pairs of coplanar conics using invariants Forsyth et al (1991), Maybank (1993). The invariants proposed by us can discriminate between two non-projectively related families of coplanar conics in which the previously used invariants give the same value.

Invariants are properties of geometric configurations which remain unchanged under an appropriate class of transformations. In fact, it has been asserted that invariance is the essential property of a shape description. Mutual invariants are properties of a set of objects that remain unchanged under a class of transformations. Cooper *et. al.* (1994) used mutual invariants in recognition of objects in aerial photographs. In a similar vein, the mutual invariants of a family of coplanar conics can be used in recognizing objects by being shape descriptors to index a model of the object from a database. Previous work with invariants from conics have focused exclusively on two coplanar conics. Often, an object will have more than two coplanar conics. This then leads to the question of using mutual invariants of more than two coplanar conics. In this paper we address that question by extending the use of invariants to recognizing objects with any number of coplanar conics.

The invariants of coplanar conics may be used as a descriptor for a planar-surface of an object. An object can, in general, have multiple planes with coplanar conics from which invariants can be obtained. The set of these invariants would comprise a description of the object. It is desirable to have as many invariants of an object as possible to index to the model, since not all or even any conics of a particular plane may be visible in a given view. To handle the case when some of the conics of a family are occluded, only a little more work is needed to calculate the invariants of a subsets of the family of conics. These invariants can then be used to give a list of candidate objects of which the visible part may be a subcomponent.

The invariants we are interested in are the *projective invariants.* A projective invariant is an attribute of an object that will remain the same under changes in pose and camera calibrations since perspective transformations are contained in the set of projective transformations.

Åström (1994) has shown some of the limitations of planar projective invariants. Although valid limitations, we take the position that they do not affect the practical application of planar invariants. First, due to the extreme nature of the series of projections needed to cause the failure of the projective planar invariants, they should be very rare in most uses. Even though each projection in the series can be realized by the pinhole camera model, the composition of the series of projection will, in general, not be realizable by a single imaging of a pinhole camera. This means for the failure to occur, the application would be of an image which contains a series of images, each one of which is contained in the previous image. This occurrence will be rare or nonexistent in many applications. Second, invariants have been used to guide the search for a match in a object database. Most of the time, a set of possible matches will be found and an additional verification step is needed to chose from the possible matches. The verification step should catch the rare mismatches due to the limitations presented by Åström.

A matrix with polynomial entries is called a *lambda-matrix.* The lambda-matrix has been used in physics and system control theory but we believe that this is the first time we are using them in the field of computer vision. We use a lambda-matrix as a tool for calculating the invariants of a family of coplanar conics. The polynomials will be in the indeterminates $\lambda_1, \lambda_2, \ldots, \lambda_n$ where n will depend on the problem at hand, and the coefficients of the polynomials will lie in the field of complex numbers. We construct 3×3 symmetric lambda-matrices where the number of indeterminates is the number of conics in the family of coplanar conics. When needed, we make a restriction to using only 3×3 symmetric with polynomials of degree at most 1. In moving to using multi-variable polynomials from the single variable polynomials, we are moving from an Euclidean Domain to an Unique Factorization Domain.

The noise in an image may affect the coefficients of the conics that are found in the image. How the noise affects the coefficients of a conic is dependent on the method used to find and fit the conic. To isolate the effects of noise on the invariants, we add simulated noise to ellipses and compare the invariants of the noisy ellipse and the original ellipse. Instead of adding noise to the coefficients of the conic, we added an percentage of error to the lengths of the major and minor axis and to the angle of rotation. This resulted in a realistic error in the fitting of a conic. A series of ellipses were randomly generated and the true invariants were calculated. Noise was then added to each axis by randomly varying the axis length in the range of $\pm$ error percentage. The angle of rotation was randomly varied. The invariants of the noisy ellipse were calculated and the absolute percent difference was recorded. The process was conducted for 500 ellipses and the average percentage of difference between the noisy ellipses and the original ellipse were calculated. It is interesting to note that not all invariants are created equal in their ability to handle noise.

2.1 Applications of Legendre Transformation to Computer Vision

The *Legendre transformation* is a well known concept in classical theoretical physics. Recently, I. Weiss of the Center for Automation Research, University of Maryland, has applied it to study problems of invariance in computer vision; his results were presented in the 1994 and 1995 ARPA Image Understanding Workshops. Most authors treat the Legendre transformation (LT) purely analytically . Only the famous Russian theoretical physicist Arnold (1978) has treated the LT from a geometric point of view; also a very general version of the LT for a hypersurface and the relationship with symplectic invariants are discussed a monograph by Hofer and Zehnder (1994).

In this paper we obtain some new geometric properties of the LT that are of potential interest in image understanding. We show that under the LT the cross ratio, and the connectedness property remain invariant. Also, we show that the LT maps a conic into a conic. As an application we develop an algorithm to detect corners in images. We also discuss the relationship of the LT with the classical Hough transform.

2.2 Matching of 3-D Polygonal Arcs

Matching of arcs is a basic problem in computer vision. It is used in many applications, e.g., industrial parts inspection, character recognition, analysis of engineering drawings, etc. The problem of finding an approximate match between short arcs and pieces of a long arc is known as the *segment matching problem.* There have been several approaches to this problem. These can be classified as global or local in nature. In the global approaches, a set of global features are obtained for each arc and we match these features for the two arcs. The 3-D generalization of the matching of polygonal arcs is of considerable interest in computer vision: the problem has applications in industrial parts inspection, motion estimation, structure determination of macromolecules from electron density maps, and in many other 3-D vision problems. Also, a polygonal approximation provides a data compression of images and several contour segmentation techniques have been proposed which have applications in the design of the vision system of a mobile robot.

There are several novel features in our approach that distinguishes it from previous works on matching of 3-D curves. Unlike the previous works, we use here unit quaternions to denote 3-D rotations that, as is well known, has the advantage (over other representations for rotations) in giving a closed form solution. Further, we proved a new result interpreting the extreme values of the distance measure for two 3-D polygonal arcs of equal lengths as the eigenvalues of a certain (well defined) 4×4 positive semidefinite matrix; the minimum value of the distance measure corresponding to the minimum eigenvalue of this matrix. It follows that the matching problem of 3-D curves can now be reduced to the purely algebraic problem of studying the eigenvalues of a certain matrix. We applied the same approach to matching 2-D polygonal arcs. We showed that a subgroup of the unit quaternions can be used to denote 2-D rotations. Using this subgroup of the unit quaternions, we proved that the minimum value of the the distance measure between two 2-D polygonal arcs is the smallest eigenvalue of the 2×2 submatrix located in the lower right quadrant of the 4×4 matrix used in 3-D matching. It follows that the matching problem of 2-D curves can now be reduced studying the eigenvalues of matrix. We then could apply standard methods of numerical linear algebra to estimate the eigenvalues and hence the matchings (up to any desired degree of accuracy). Our algorithm is practical to implement and we have carried out extensive implementations.

3 Personnel Supported

Doug Heisterkamp, Ph.D. student, 12 months. (As per the conditions of an AASERT grant, the student is a US citizen.)

4 Interactions/Transitions

The results from our effort could be used in developing technology for automatic target recognition.

5 Presentations in Conferences

1. April, 1996, IEEE International Conference on Robotics and Automation, Minneapolis, Minnesota. Oral presentation.

2. August, 1996, International CΛonference on Pattern Recognition (ICPR), Vienna, Austria. Oral presentation.

5.1 Interaction with Air Force Laboratories

The PI gave a presentation of his work at the Wright-Patterson Air Force Laboratory, Dayton, OH in 1995. He gave two talks on successive days and also held detailed discussions with some staff members including V. Velten and J. Leonard.

The PI was invited to participate in the Smart Sensors Workshop held in 1996 at the Air Force Laboratory at the Kirtland Air Force Base in Albuquerque, NM. He also gave a poster presentation at the workshop. During the workshop, he benefited greatly by interacting with the other participants including several Air Force staff.

6 Technical Publications

6.1 Journal Papers

1. D. Heisterkamp and P. Bhattacharya, "Matching of Three-Dimensional Polygonal Arcs," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 19, no. 1, pp. 68-73, (1997).

2. D. Heisterkamp and P. Bhattacharya, "Invariants of Families of Coplanar Conics, and their Applications to Object Recognition," *Journal of Mathematical Imaging and Vision*, vol. 7, pp. 253-267, (1997).

3. D. Heisterkamp and P. Bhattacharya, "Matching of 2-D Polygonal Arcs Using a Subgroup of the Unit Quaternions, *Computer Vision and Image Understanding*," accepted for publication.

4. D. Heisterkamp and P. Bhattacharya, "Geometric Properties of Legendre Transformations," *Computer Vision and Image Understanding*, tentatively accepted for publication, subject to revision.

6.2 Refereed Conference Papers

5. D. Heisterkamp and P. Bhattacharya, "Matching of 3-D Curves", *Proc. IEEE Internat. Conf. Robotics and Automation*, Minneapolis, April, 1996, pp. 3490–3495.

6. D. Heisterkamp and P. Bhattacharya, "Invariants of a Family of Coplanar Conics and Object Recognition," *Proc. 13th Internat. Conf. Pattern Recognition* (ICPR'96), Vienna, Austria, August, 1996, Computer Vision Track, pp. 677-681.

7 Patent disclosures

None.

8 Honors/Awards

1. National Lecturer of the Association of Computing Machinery (ACM) during 1996-99.
2. Distinguished Visitor of the IEEE Computer Society during 1996-99.
3. Award for highest involvement from the Nebraska chapter of the IEEE Computer Society, 1996.
4. Certificate for being an outstanding member from the Nebraska chapter of the IEEE Computer Society, 1996.
5. Editorial board member of the IEEE Computer Society Press during 1996-98.
6. Editorial board member of "Pattern Recognition" during 1994-present.
7. Elected the Chairman of the IEEE Computer Society, Nebraska Chapter for 1995-96, and re-elected for 1996-97.